Bell's inequalities

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Introduction

By 1935, it was already recognized that the predictions of quantum mechanics (QM) are probabilistic. In their famous paper of 1935 Albert Einstein, Boris Podolsky and Nathan Rosen presented a scenario that, in their view, indicated that quantum particles, like electrons and photons, must carry physical properties or attributes not included in QM, and the uncertainties in predictions of QM were due to ignorance of these properties, later termed *hidden variables*.

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Spin correlations in a singlet state

Consider a spin $\frac{1}{2}$ particle. The spin operator has the form

$$\vec{S} = \frac{1}{2}\vec{\sigma},$$

where we have assumed $\hbar = 1$.

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Let $\hat{a}$ denote the unit vector in the direction of the inhomogeneous magnetic field. Instead of spin projection onto it, i.e., $\hat{a} \cdot \vec{S}$ it is more convenient to use the observable

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which has the eigenvalues ± 1 instead of $\pm\frac{1}{2}$.

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If we choose $\hat{a} = \hat{e}_3$ then we will obtain

$$\not{e}_3 = 2\hat{e}_3 \cdot \vec{S} = \sigma_3,$$

with the eigenvectors $\not{e}_3 \left|\pm\right\rangle = \pm \left|\pm\right\rangle$ of the following form

$$\left|+\right\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad \left|-\right\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Obviously

$$\left\langle+\right| = \begin{pmatrix} 1 & 0 \end{pmatrix}, \qquad \left\langle-\right| = \begin{pmatrix} 0 & 1 \end{pmatrix}.$$

Now, recall that

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and calculate

$$\not{a} = \hat{a} \cdot \vec{\sigma} = \begin{pmatrix} a_3 & a_1 - ia_2 \\ a_1 + ia_2 & -a_3 \end{pmatrix}.$$

Let us find the eigenvalues of $\not{a}$. To this end we have to solve the equation

$$\begin{vmatrix} a_3 - \lambda & a_1 - ia_2 \\ a_1 + ia_2 & -a_3 - \lambda \end{vmatrix} = \lambda^2 - a_3^2 - a_1^2 - a_2^2 = 0.$$

As $\hat{a}$ is the unit vector, we get $\lambda = \pm 1$.

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Thus, the eigenequation of $\not{a}$ has the form

$$\not{a}\,|\hat{a}\pm\rangle = \pm\,|\hat{a}\pm\rangle\,.$$

Vector $\hat{a}$ can be obtained from vector $\hat{e}_3$ by a rotation of angle $\vec{\theta} = \theta\hat{\theta}$, with $\hat{\theta}$ being a unit vector parallel to $\hat{e}_3 \times \hat{a}$, which determines the direction of the rotation axis. Hence, we have

$$|\hat{a}\pm\rangle = e^{-i\vec{\theta}\cdot\vec{S}}\,|\pm\rangle\,,$$

where

$$e^{-i\vec{\theta}\cdot\vec{S}} S_3 e^{i\vec{\theta}\cdot\vec{S}} = \hat{a}\cdot\vec{S}.$$

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Since it can be shown that

$$e^{-i\vec{\theta}\cdot\vec{S}} = \cos\frac{\theta}{2} - i\hat{\theta}\cdot\vec{\sigma}\sin\frac{\theta}{2},$$

we have

$$|\hat{a}\pm\rangle = \left(\cos\frac{\theta}{2} - i\hat{\theta}\cdot\vec{\sigma}\sin\frac{\theta}{2}\right)|\pm\rangle\,.$$

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Let us calculate matrix elements of $\not{a}$ in the spin eigenvector basis

$$\langle +|\not{a}|+\rangle = (1\ \ 0)\begin{pmatrix} a_3 & a_1 - ia_2 \\ a_1 + ia_2 & -a_3 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = (1\ \ 0)\begin{pmatrix} a_3 \\ a_1 + ia_2 \end{pmatrix}$$
$$= a_3,$$
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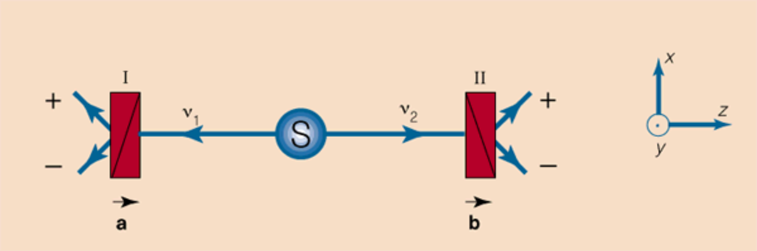

Figure: Simultaneous spin measurements on particle pairs $(1) + (2)$. S is the particle source, and $\vec{a}$ and $\vec{b}$ are field directions of the Stern–Gerlach magnets.

Now we consider the combination of two different spin-$\frac{1}{2}$ systems. A system of basis vectors is

$$|(1)+\rangle \otimes |(2)+\rangle\,, \quad |(1)-\rangle \otimes |(2)-\rangle\,,$$
$$|(1)+\rangle \otimes |(2)-\rangle\,, \quad |(1)-\rangle \otimes |(2)+\rangle\,,$$

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where (1) and (2) refer to the first and second particle, respectively, and $+$ and $-$ specifies the z component of the spin.

We can also use the Stern–Gerlach device that measures the spin component of the first particle along $\hat{a}$ and the spin component of the second particle along $\hat{b}$.

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Then we take the following basis system

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with two arbitrarily chosen vectors unit vectors $\hat{a}$ and $\hat{b}$.
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$$\not{a} \otimes \mathbb{I} = 2\hat{a} \cdot \vec{S} \otimes \mathbb{I} \quad \text{and} \quad \mathbb{I} \otimes \not{b} = \mathbb{I} \otimes 2\hat{b} \cdot \vec{S},$$

which are $2 \times$ spin component of particle (1) along $\hat{a}$ and $2 \times$ spin component of particle (2) along $\hat{b}$, respectively.

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They act on our basis vectors $|\hat{a}\alpha\rangle \otimes |\hat{b}\beta\rangle$, $\alpha, \beta = \pm 1$, in the following way

$$\not{a} \otimes \mathbb{I}\, |\hat{a}\alpha\rangle \otimes |\hat{b}\beta\rangle = \not{a}\, |\hat{a}\alpha\rangle \otimes \mathbb{I}\, |\hat{b}\beta\rangle = \alpha\, |\hat{a}\alpha\rangle \otimes |\hat{b}\beta\rangle$$
$$\mathbb{I} \otimes \not{b}\, |\hat{a}\alpha\rangle \otimes |\hat{b}\beta\rangle = \mathbb{I}\, |\hat{a}\alpha\rangle \otimes \not{b}\, |\hat{b}\beta\rangle = \beta\, |\hat{a}\alpha\rangle \otimes |\hat{b}\beta\rangle .$$

The two observables commute and our basis vectors are simultaneously eigenvectors of both of them with eigenvalues $+1$ or -1. Indeed, let us calculate

$$[\not{a} \otimes \mathbb{I}, \mathbb{I} \otimes \not{b}] = (\not{a} \otimes \mathbb{I})(\mathbb{I} \otimes \not{b}) - (\mathbb{I} \otimes \not{b})(\not{a} \otimes \mathbb{I})$$
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Thus $\not{a} \otimes \mathbb{I}$ and $\mathbb{I} \otimes \not{b}$ can be measured simultaneously.

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The measurement of $\not{a} \otimes \mathbb{I}$ in the basis states

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will always yield $+1, -1, +1, -1$, and the simultaneous measurement of $\mathbb{I} \otimes \not{b}$ will yield $+1, -1, -1, +1$.

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For such simultaneous measurement we can in addition define a *spin correlation observable*, which is by definition the product of the values obtained in a single measurement of both $\not{a} \otimes \mathbb{I}$ and $\mathbb{I} \otimes \not{b}$.

This spin correlation observable is described by the operator

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whose eigenvalues values are $+1$ for the first two vectors and -1 for the last two vectors of the above basis.

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The simultaneous spin measurements on two-particle systems with two Stern–Gerlach devices are possible only if the two particles of each pair are spatially separated and each particle moves along a certain fixed axis, as shown in the Figure below.

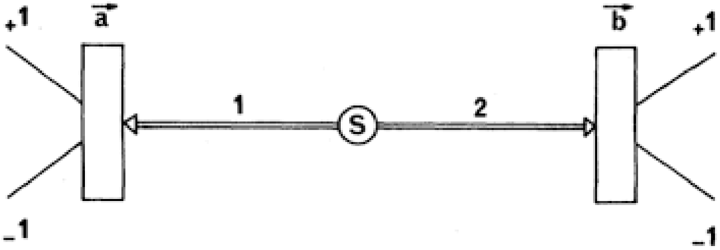

Figure: Simultaneous spin measurements on particle pairs $(1) + (2)$.

A particle source emits pairs of particles, one pair at a time, such that particle (1) is always emitted to the left, and particle (2) is always emitted to the right.

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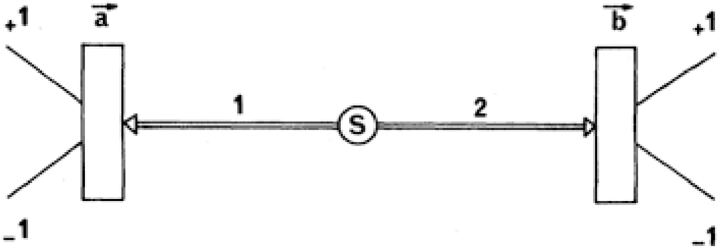

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Then a Stern–Gerlach device with inhomegenous magnetic field along some direction $\hat{a}$, perpendicular to the beam, may be applied to the left beam and another Stern–Gerlach device with field direction $\hat{b}$ may be applied to the right beam. Each device has two counters, one at a position $+1$ and and the other at a position -1. Since the particles (1) and (2) are emitted pairwise by the source, the two particles of a single pair pass the two Stern–Gerlach magnets an arrive at two of the four counters almost simultaneously.

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Therefore, a click of the $+1$ counter on the left and -1 counter on the right means a simultaneous measurement of $\not{a} \otimes \mathbb{I}$ with the result $+1$ and of $\mathbb{I} \otimes \not{b}$ with the result -1. The spin correlation observable $\not{a} \otimes \not{b}$ has then the value $(+1)(-1) = -1$.

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This kind of measurement is repeated N times, with $N \gg 1$, and the following numbers are recorded:

- the number N_{++} of simultaneous clicks of $+1$ counter on the left and $+1$ counter on the right,
- the number N_{+-} of simultaneous clicks of $+1$ counter on the left and -1 counter on the right,

the numbers N_{-+} and N_{--} are defined similarly.

The measured *average values* for the observables $\not{a} \otimes \mathbb{I}$, $\mathbb{I} \otimes \not{b}$ and $\not{a} \otimes \not{b}$, which we denote, respectively, by $E_1(\hat{a})$, $E_2(\hat{b})$ and $E(\hat{a}, \hat{b})$ are the following:

$$E_1(\hat{a}) = \frac{1}{N}(N_{++} + N_{+-} - N_{-+} - N_{--}),$$

$$E_2(\hat{b}) = \frac{1}{N}(N_{++} - N_{+-} + N_{-+} - N_{--}),$$

$$E(\hat{a}, \hat{b}) = \frac{1}{N}(N_{++} - N_{+-} - N_{-+} + N_{--}),$$

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According to quantum mechanics, these measured average values should coincide with the *expectation values* of corresponding observables in the common spin state of the particle pairs emitted by the source.

The combination of two spin-$\frac{1}{2}$ systems may lead to the total spin value

$$s = \left|\frac{1}{2} - \frac{1}{2}\right|, ..., \frac{1}{2} + \frac{1}{2},$$

i.e., $s = 0$ or $s = 1$.

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We will assume here that the particle pairs emitted by the source have total spin 0, and are therefore in the asymmetric singlet state

$$|\phi\rangle = \frac{1}{\sqrt{2}}\left(|(1)+\rangle \otimes |(2)-\rangle - |(1)-\rangle \otimes |(2)+\rangle\right)$$
$$\equiv \frac{1}{\sqrt{2}}\left(|+\rangle \otimes |-\rangle - |-\rangle \otimes |+\rangle\right).$$

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$$s = \left|\frac{1}{2} - \frac{1}{2}\right|, \ldots, \frac{1}{2} + \frac{1}{2},$$

i.e., $s = 0$ or $s = 1$.
We will assume here that the particle pairs emitted by the source have total spin 0, and are therefore in the asymmetric singlet state

$$\begin{aligned} |\phi\rangle &= \frac{1}{\sqrt{2}}\left(|(1)+\rangle \otimes |(2)-\rangle - |(1)-\rangle \otimes |(2)+\rangle\right) \\ &\equiv \frac{1}{\sqrt{2}}\left(|+\rangle \otimes |-\rangle - |-\rangle \otimes |+\rangle\right). \end{aligned}$$

Spin correlations in a singlet state

A source emitting particle pairs in the spin state $|\phi\rangle$ might contain, e.g., a large number of unstable compounds of two particles (1) and (2) at rest, and therefore after the decay, due to momentum conservation, particles (1) and (2) move always in opposite directions. However, while moving away, they are still in the common spin state $|\phi\rangle$.

We can now calculate the corresponding QM expectation values.

$$
\begin{aligned}
\langle\phi|\not{a}\otimes\mathbb{I}|\phi\rangle &= \frac{1}{2}(\langle+|\otimes\langle-|-\langle-|\otimes\langle+|)\not{a}\otimes\mathbb{I}(|+\rangle\otimes|-\rangle-|-\rangle\otimes|+\rangle) \\
&= \frac{1}{2}(\langle+|\otimes\langle-|-\langle-|\otimes\langle+|)(\not{a}\,|+\rangle\otimes|-\rangle-\not{a}\,|-\rangle\otimes|+\rangle) \\
&= \frac{1}{2}(\langle+|\not{a}|+\rangle\langle-|-\rangle-\langle+|\not{a}|-\rangle\langle-|+\rangle-\langle-|\not{a}|+\rangle\langle+|-\rangle \\
&\qquad +\langle-|\not{a}|-\rangle\langle+|+\rangle) = \frac{1}{2}(a_3-a_3) = 0,
\end{aligned}
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where we have used $\langle+|\not{a}|+\rangle = a_3$ and $\langle-|\not{a}|-\rangle = -a_3$.

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Spin correlations in a singlet state

Using $\langle +|\not{a}|+\rangle = a_3$, $\langle -|\not{a}|-\rangle = -a_3$, $\langle +|\not{a}|-\rangle = a_1 - ia_2$ and $\langle -|\not{a}|+\rangle = a_1 + ia_2$, and analogously for $\not{b}$, we will get

$$\begin{aligned}\langle\phi|\mathbb{I}\otimes\not{b}|\phi\rangle &= \frac{1}{2}(\langle +|\otimes\langle -| - \langle -|\otimes\langle +|)\mathbb{I}\otimes\not{b}(|+\rangle\otimes|-\rangle - |-\rangle\otimes|+\rangle)\\ &= \frac{1}{2}(\langle +|\otimes\langle -| - \langle -|\otimes\langle +|)(|+\rangle\otimes\not{b}\,|-\rangle - |-\rangle\otimes\not{b}\,|+\rangle)\\ &= \frac{1}{2}(\langle +|+\rangle\langle -|\not{b}|-\rangle - \langle +|-\rangle\langle -|\not{b}|+\rangle - \langle -|+\rangle\langle +|\not{b}|-\rangle\\ &\qquad + \langle -|-\rangle\langle +|\not{b}|+\rangle) = \frac{1}{2}(-b_3 + b_3) = 0.\end{aligned}$$

Similarly

$$\begin{aligned}\langle\phi|\not{a}\otimes\not{b}|\phi\rangle &= \frac{1}{2}(\langle +|\not{a}|+\rangle\langle -|\not{b}|-\rangle - \langle +|\not{a}|-\rangle\langle -|\not{b}|+\rangle - \langle -|\not{a}|+\rangle\langle +|\not{b}|-\rangle\\ &\qquad + \langle -|\not{a}|-\rangle\langle +|\not{b}|+\rangle) = \frac{1}{2}\,(a_3(-b_3) - (a_1 - ia_2)(b_1 + ib_2)\\ &\qquad - (a_1 + ia_2)(b_1 - ib_2) + (-a_3)b_3) = -\hat{a}\cdot\hat{b}.\end{aligned}$$

Spin correlations in a singlet state

The QM predictions for the expectation values $\langle\phi|\not{a}\otimes\mathbb{I}|\phi\rangle$, $\langle\phi|\mathbb{I}\otimes\not{b}|\phi\rangle$ and $\langle\phi|\not{a}\otimes\not{b}|\phi\rangle$ hold obviously for a large number of single particle pair spin measurements.

Compare the QM prediction for

$$\langle\phi|\not{a}\otimes\mathbb{I}|\phi\rangle = 0$$

with the measured average value

$$E_1(\hat{a}) = \frac{1}{N}(N_{++} + N_{+-} - N_{-+} - N_{--}).$$

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Spin correlations in a singlet state

The same conclusion can be derived for particle (2) if we compare the QM prediction for $\langle\phi|\mathbb{I}\otimes \not{b}|\phi\rangle$ with the measured average value of

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Spin correlations in a singlet state

Then, remembering that $N = N_{++} + N_{+-} + N_{-+} + N_{--}$ we will get the following condition

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and, since both N_{++} and N_{--} are positive, we see that

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According to the QM prediction

$$E(\hat{a}, \hat{a}) = -1$$

spin components of two particles (1) and (2) along a fixed direction $\hat{a}$ are always opposite to each other.

Instead of directly measuring $\not{a} \otimes \mathbb{I}$ on particle (1) itself, we can equally well determine its spin component along $\hat{a}$ by measuring $\mathbb{I} \otimes \not{a}$ on particle (2) and multiplying the result by -1.

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Therefore, it seems quite *natural* to imagine that a single particle (1) does not *get* a definite spin component along $\hat{a}$ during a measurement of $\not{a} \otimes \mathbb{I}$, but rather *has* a definite value of it, either $+1$ or -1, prior to and independent of any measurement.

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Assume that prior to and independent of any measurement every single particle (1) possesses a definite value $v(\hat{a})$, of either $+1$ or -1, for the components of its spin, at least along all possible directions $\hat{a}$ orthogonal to the beam.

These values are just *uncovered*, rather than *produced*, if the actual spin measurement is performed. They may be visualized as hidden labels, either $+1$ or -1, attached to every single particle (1) for every possible direction $\hat{a}$. The same argument applies obviously to all particles (2).

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Without this assumption it seems quite difficult to understand the perfect anticorrelation $\langle\phi|\not{a}\otimes\not{a}|\phi\rangle = -1$ for simultaneous measurements of $\not{a}\otimes\mathbb{I}$ on particle (1) and $\mathbb{I}\otimes\not{a}$ on particle (2).

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For if the value of $\not{a}\otimes\mathbb{I}$ was really undetermined until it is actually measured on particle (1), it would appear impossible for particle (2), which may be very far away, to *get informed* about this value, in order to be able to *choose* just the opposite value.

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Consider a very large number N of particle pairs in the spin singlet state

$$|\phi\rangle = \frac{1}{\sqrt{2}}(|+\rangle \otimes |-\rangle - |-\rangle \otimes |+\rangle)$$

and four arbitrarily chosen directions $\hat{a}$, $\hat{b}$, $\hat{c}$ and $\hat{d}$ in the plane orthogonal to the two beams of particles produced by the source. Denote by $v_i(\hat{a})$ and $v_i(\hat{d})$ the *hidden* predetermined values of the spin components along $\hat{a}$ and $\hat{d}$ of particle (1) in the i-th pair, and by $w_i(\hat{b})$ and $w_i(\hat{c})$ the *hidden* predetermined values of the spin components along $\hat{b}$ and $\hat{c}$ of particle (2) in the same pair.

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Then, if the spin component of particle (1) along $\hat{a}$ and the spin component of particle (2) along $\hat{b}$ are measured simultaneously for all N pairs the average spin correlation value becomes

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But we could have chosen other directions, e.g., $\hat{d}$ in the left and $\hat{c}$ in the right beam, for the orientation of the two Stern–Gerlach devices.

If such experiment had been performed instead with *the same N* particle pairs, it would have *uncovered* the spin components $v_i(\hat{d})$ and $w_i(\hat{c})$ and the observed spin correlation average would have been

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Bell's inequalities

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It seems reasonable to assume that an experimental determination of $E(\hat{d}, \hat{c})$ *would have produced the same result* if it had been performed on any of the four sets of N particle pairs.

Therefore $E(\hat{a}, \hat{b})$, $E(\hat{d}, \hat{c})$, $E(\hat{a}, \hat{c})$ and $E(\hat{d}, \hat{b})$ may be considered as quantities characteristic of the N particle pairs set and their preparation, and are not dependent on which particular experiment is actually performed.

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To this end let us first show that

$$v_i(\hat{a})(w_i(\hat{b}) + w_i(\hat{c})) + v_i(\hat{d})(w_i(\hat{b}) - w_i(\hat{c})) = \pm 2,$$

$i = 1, 2, ..., N$.

Proof. As w_i is either $+1$ or -1, the first bracket is either $+2, 0$ or -2.

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$$v_i(\hat{a})(w_i(\hat{b}) + w_i(\hat{c})) + v_i(\hat{d})(w_i(\hat{b}) - w_i(\hat{c})) = \pm 2,$$

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$$-2N \leq \sum_{i=1}^{N} \left[v_i(\hat{a})(w_i(\hat{b}) + w_i(\hat{c})) + v_i(\hat{d})(w_i(\hat{b}) - w_i(\hat{c})) \right] \leq 2N,$$

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and dividing this by N we obtain

$$\left|\frac{1}{N}\sum_{i=1}^{N} v_i(\hat{a})w_i(\hat{b}) + \frac{1}{N}\sum_{i=1}^{N} v_i(\hat{a})w_i(\hat{c}) + \frac{1}{N}\sum_{i=1}^{N} v_i(\hat{d})w_i(\hat{b}) - \frac{1}{N}\sum_{i=1}^{N} v_i(\hat{d})w_i(\hat{c})\right| \leq 2.$$

Thus, we obtain the inequality

$$\left|E(\hat{a},\hat{b}) + E(\hat{a},\hat{c}) + E(\hat{d},\hat{b}) - E(\hat{d},\hat{c})\right| \leq 2,$$

which is the most famous and experimentally most useful of a series of similar inequalities known as *Bell's inequalities.*

Let us now check if the QM prediction

$$E(\hat{a},\hat{b}) = \langle\phi|\not{a}\otimes\not{b}|\phi\rangle = -\hat{a}\cdot\hat{b}$$

satisfies the above Bell's inequality.

$$\begin{aligned}
&|-\hat{a}\cdot\hat{b}-\hat{a}\cdot\hat{c}-\hat{d}\cdot\hat{b}+\hat{d}\cdot\hat{c}| = |\hat{a}\cdot\hat{b}+\hat{a}\cdot\hat{c}+\hat{d}\cdot\hat{b}-\hat{d}\cdot\hat{c}|\\
&= |\hat{a}\cdot(\hat{b}+\hat{c})+\hat{d}\cdot(\hat{b}-\hat{c})| \le |\hat{a}||\hat{b}+\hat{c}|+|\hat{d}||\hat{b}-\hat{c}|\\
&= |\hat{b}+\hat{c}|+|\hat{b}-\hat{c}| = \sqrt{(\hat{b}+\hat{c})^2}+\sqrt{(\hat{b}-\hat{c})^2}\\
&= \sqrt{2+2\cos\theta}+\sqrt{2-2\cos\theta},
\end{aligned}$$

with θ being the angle between $\hat{b}$ and $\hat{c}$, $\hat{b}\cdot\hat{c}=\cos\theta$, $\theta\in[0,\pi]$.

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$$\begin{aligned}
&| - \hat{a} \cdot \hat{b} - \hat{a} \cdot \hat{c} - \hat{d} \cdot \hat{b} + \hat{d} \cdot \hat{c}| = |\hat{a} \cdot \hat{b} + \hat{a} \cdot \hat{c} + \hat{d} \cdot \hat{b} - \hat{d} \cdot \hat{c}| \\
&= |\hat{a} \cdot (\hat{b} + \hat{c}) + \hat{d} \cdot (\hat{b} - \hat{c})| \leq |\hat{a}||\hat{b} + \hat{c}| + |\hat{d}||\hat{b} - \hat{c}| \\
&= |\hat{b} + \hat{c}| + |\hat{b} - \hat{c}| = \sqrt{(\hat{b} + \hat{c})^2} + \sqrt{(\hat{b} - \hat{c})^2} \\
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Bell's inequalities

Denote the expression on the right hand side of our inequality by

$$f(\theta) = \sqrt{2 + 2\cos\theta} + \sqrt{2 - 2\cos\theta} = 2\sqrt{\frac{1 + \cos\theta}{2}} + 2\sqrt{\frac{1 - \cos\theta}{2}},$$

which for $\theta \in [0, \pi]$ can be written as

$$f(\theta) = 2\cos\frac{\theta}{2} + 2\sin\frac{\theta}{2}.$$

Let us find the maximum of $f(\theta)$.

$$f'(\theta) = -\sin\frac{\theta}{2} + \cos\frac{\theta}{2} = 0 \quad \Leftrightarrow \quad \frac{\theta}{2} = \frac{\pi}{4}.$$

Thus $f(\theta) = 0$ for $\theta = \frac{\pi}{2}$. Calculate

$$f''(\theta) = -\frac{1}{2}\cos\frac{\theta}{2} - \frac{1}{2}\sin\frac{\theta}{2}\Big|_{\theta=\frac{\pi}{2}} = -2\frac{\sqrt{2}}{2} - 2\frac{\sqrt{2}}{2} = -2\sqrt{2} < 0.$$

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Thus, $f(\theta)$ has the maximum at $\theta = \frac{\pi}{2}$ equal to

$$f(\theta) = \sqrt{2 + 2\cos\frac{\pi}{2}} + \sqrt{2 - 2\cos\frac{\pi}{2}} = 2\sqrt{2}$$

and the QM prediction for our inequality is the following

$$\left| E(\hat{a}, \hat{b}) + E(\hat{a}, \hat{c}) + E(\hat{d}, \hat{b}) - E(\hat{d}, \hat{c}) \right| \leq 2\sqrt{2}.$$

The Bell inequality becomes equality, i.e., it is maximally violated by the QM prediction, if

1. $\hat{a}$ and $\hat{b} + \hat{c}$, and $\hat{d}$ and $\hat{b} - \hat{c}$ are parallel,
2. $\hat{a}$ and $\hat{b} + \hat{c}$, and $\hat{d}$ and $\hat{b} - \hat{c}$ are antiparallel.

These configurations are depicted in the Figure below.

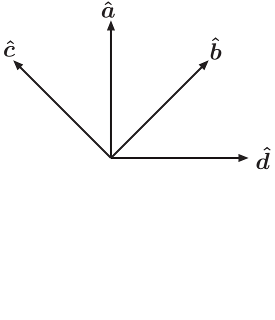

Figure: Magnetic field configurations of the Stern–Gerlach devices for which Bell's inequality is maximally violated.

The equality

$$\left| E(\hat{a},\hat{b}) + E(\hat{a},\hat{c}) + E(\hat{d},\hat{b}) - E(\hat{d},\hat{c}) \right| = 2\sqrt{2}$$

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The best way is to solve the conflict *empirically*, by performing our experiment four times with the Stern–Gerlach devices oriented according to one of the configurations depicted in the Figure on a previous slide to determine the four averages $E(\hat{a}, \hat{b})$, $E(\hat{a}, \hat{c})$, $E(\hat{d}, \hat{b})$ and $E(\hat{d}, \hat{c})$, each time with many single particle pairs.

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This is not as easy in practice as one might imagine, however.

First of all it is difficult to prepare pairs of spin-$\frac{1}{2}$ particles in a spin singlet state $|\phi\rangle = \frac{1}{\sqrt{2}}(|+\rangle \otimes |-\rangle - |-\rangle \otimes |+\rangle)$.

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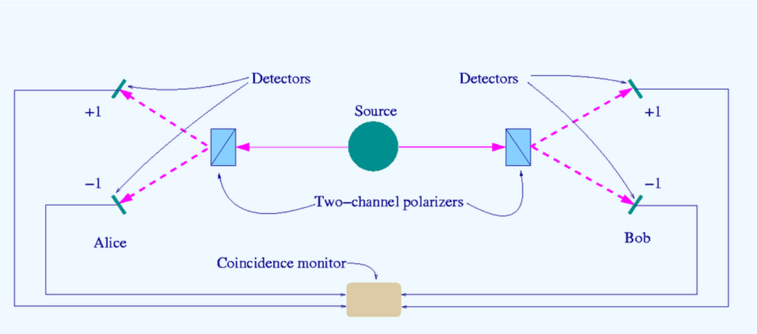
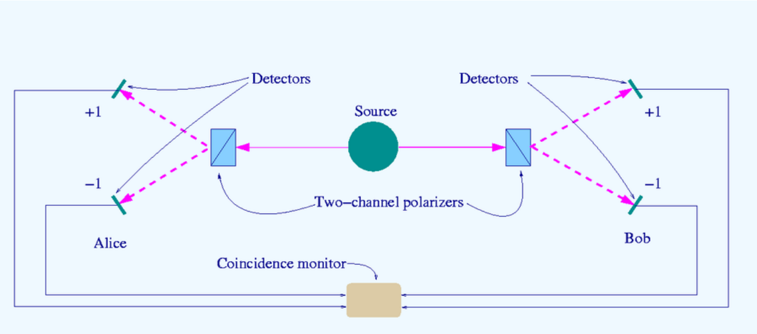


Figure: Scheme of a photon analyzer for tests of Bell's inequalities.

In this case, the simultaneous spin measurements discussed till now are replaced by simultaneous measurements of the transverse linear polarizations of the two emitted photons along arbitrarily chosen directions $\hat{a}$ and $\hat{b}$.



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For the photon pairs emitted in cascade transitions, these polarizations are correlated in much the same way as the spin components of spin-$\frac{1}{2}$ particle pairs in the considered singlet state $|\phi\rangle$.

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The hypothesis that two spin-$\frac{1}{2}$ particles *have* definite spin components $v_i(\hat{a})$ and $w_i(\hat{b})$ prior to the measurement seems natural only from the classical point of view, in which we visualize the particle pair as consisting of two separate particles.

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However, the QM state vector

$$|\phi\rangle = \frac{1}{\sqrt{2}}\left(|(1)+\rangle \otimes |(2)-\rangle - |(1)-\rangle \otimes |(2)+\rangle\right)$$

does not describe a state with separate single-particle properties.
Such states would be described by any one of the basis vectors

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The QM state vector $|\phi\rangle$ describes a new entity, an *indivisible whole*, a single object whose constituent particles (1) and (2) are not definable until a measurement is made that prepares the direct product states $|(1)+\rangle \otimes |(2)+\rangle$, $|(1)-\rangle \otimes |(2)-\rangle$, $|(1)+\rangle \otimes |(2)-\rangle$ and $|(1)-\rangle \otimes |(2)+\rangle$, or their mixtures.

Being a state with the total spin 0, $|\phi\rangle$ does not have single-particle properties. And as a *pure* QM state it cannot be subdivided.

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In classical physics, the building blocks of a composite system are its constituents. In QM the building blocks are subspaces of the Hilbert space of physical states. They can be any kind of subspaces, not only the one-dimensional subspaces spanned by any of the direct products listed above. The state $|\phi\rangle$ is one of the multitude of arbitrary linear combinations of them.

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In classical physics, the building blocks of a composite system are its constituents. In QM the building blocks are subspaces of the Hilbert space of physical states. They can be any kind of subspaces, not only the one-dimensional subspaces spanned by any of the direct products listed above. The state $|\phi\rangle$ is one of the multitude of arbitrary linear combinations of them.

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Historical remark

In their paper of 1935 Einstein, Podolsky and Rosen (EPR) proposed a thought experiment which was supposed to prove that QM is an *incomplete* theory. They used essentially the same *paradox* as the one discussed above.

They considered a pair of point particles with, for simplicity, one-dimensional coordinates x_1 and x_2 in the improper, i.e., unnormalized, state $|\phi\rangle$ described by the wave function

$$\langle x_1 x_2 | \phi \rangle = \delta(x_1 - x_2 - a).$$

The Fourier transform of it has the form

$$\langle p_1 p_2 | \phi \rangle = \frac{1}{2\pi} \int e^{-i(p_1 x_1 + p_2 x_2)} \langle x_1 x_2 | \phi \rangle \, \mathrm{d}x_1 \mathrm{d}x_2$$

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