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$ee4f\gamma$ —A program for $e^+e^- \rightarrow 4f, 4f\gamma$ with nonzero fermion masses ^{☆,☆☆}

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Abstract

A computer program $ee4f\gamma$ for calculating cross-sections of any four fermion final state of e^+e^- -annihilation at high energy and the corresponding bremsstrahlung reaction that is possible in the framework of the Standard Model is presented. As the fermion masses are arbitrary, the cross-sections for channels that do not contain e^+ and/or e^- in the final state can be computed without any collinear cut, the on-shell top quark production can be studied and the Higgs boson exchange can be incorporated in a consistent way. The program can be used as a Monte Carlo generator of unweighted events as well.

Program summary

Title of program: $ee4f\gamma$

Version: 1.0 (February 2004)

Catalogue identifier: ADTQ

Program summary URL: <http://cpc.cs.qub.ac.uk/summaries/ADTQ>

Program obtainable from: CPC Program Library, Queen's University of Belfast, N. Ireland

Licensing provisions: none

Computers: all

Operating systems: Unix/Linux

Programming language used: FORTRAN 90

CPC Program Library subprograms used: RANLUX, ACPR RANLUX 79 (1994) 111—a random number generator

Memory required to execute with typical data: 4.0 Mb

No. of bits in a word: 32

No. of bytes in distributed program, including test data, etc.: 364 490

No. of lines in distributed program, including test data, etc.: 45 278

Distribution format: tar gzip file

[☆] This paper and its associated computer program are available via the Computer Physics Communications homepage on ScienceDirect (<http://www.sciencedirect.com/science/journal/00104655>).

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Nature of physical problem: Description of all $e^+e^- \rightarrow 4$ fermions and corresponding bremsstrahlung reactions that are possible in the Standard Model (SM) to lowest order and with nonzero fermion masses at center of mass energies typical for next generation linear colliders. Such reactions are relevant, typically, for W -pair or intermediate mass Higgs boson production and decay.

Method of solution: Matrix elements are calculated with the helicity amplitude method. The phase space integration is performed numerically utilizing a multi-channel Monte Carlo method.

Restrictions on complexity of the problem: No higher order effects are taken into account, except for assuming the fine structure constant and the strong coupling at the appropriate scale and partial summation of those one particle irreducible loop corrections which are inducing the (fixed) finite widths of unstable particles.

Typical running time: The running time depends strongly on a selected channel and desired precision of the result. The results of the appended test run have been obtained on a 800 MHz Pentium III processor with the use of Absoft FORTRAN 90 compiler in about 9 minutes. In order to obtain a precision level below one per mille a few million calls to the integrand are required. This results in less than one hour running time for the fastest channel $e^+e^- \rightarrow \nu_\mu \bar{\nu}_\mu \nu_\tau \bar{\nu}_\tau$ and more than 100 hours running time for the slowest channel $e^+e^- \rightarrow e^+e^-e^+e^-$. However, the typical running time for channels that are relevant for the W -pair or Higgs boson production is several hours.

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1. Introduction

Precise tests of the Standard Model (SM) that will become possible at future high luminosity e^+e^- linear colliders like TESLA [1], the NLC [2], or the JLC [3] require adequate high precision of theoretical predictions. To meet this requirement is a challenge, as the complexity of perturbative calculation increases while going to higher energy and higher luminosity. The spectrum of possible reactions involves among others the four fermion channels with possible real photon emission. These reactions, which are of particular relevance for W -pair or intermediate mass Higgs boson production and decay, receive contributions from several to about one thousand Feynman diagrams already at the tree level of the SM.

In this article, a technical documentation of a numerical program `ee4fγ` is presented which allows for computer simulation of all the four fermion reactions

$$e^+(p_1) + e^-(p_2) \rightarrow f_1(p_3) + \bar{f}_2(p_4) + f_3(p_5) + \bar{f}_4(p_6), \quad (1)$$

and the corresponding bremsstrahlung reactions

$$e^+(p_1) + e^-(p_2) \rightarrow f_1(p_3) + \bar{f}_2(p_4) + f_3(p_5) + \bar{f}_4(p_6) + \gamma(p_7), \quad (2)$$

where f_i , $i = 1, 2, 3, 4$, make up a four fermion final state possible in the SM and the particle momenta have been indicated in parentheses. The physics contents of the program has been discussed in earlier works [4–6], where the details on the underlying physics, the method of calculation and physical results obtained with `ee4fγ` have been presented.

In the approximation of massless fermions, all possible four fermion channels for $e^+e^- \rightarrow 4f$ have been investigated in [7] (`EXCALIBUR`) and those for $e^+e^- \rightarrow 4f\gamma$ in [8] (`RACCOONWW`). `ee4fγ` makes possible computations with nonzero fermion masses. Keeping nonzero fermion masses will be important in cases when predictions at the 1% accuracy level are required [4–6]. Finite masses also provide a natural regularization of distributions which become singular in the massless limit. Massive calculations thus provide reliable benchmarks for massless calculations with cuts. The latter are much simpler and hence much faster than calculations with massive codes.

`ee4fγ` may be considered as a building block (the soft plus hard bremsstrahlung part) of a complete $O(\alpha)$ calculation of the processes $e^+e^- \rightarrow 4f$. Such calculations have been attempted in [9] (see also [10]) and have been

completed in the double pole approximation for W -pair production [11] (RaccoonWW) and [12] (KORALW/YFSWW) (see also [13] (EEWW)). The double pole approximation incorporates the one-loop corrections for on-shell W -pair production [14] and the subsequent decay of W 's into fermion-pairs [15].

Several existing programs, some of them being general purpose packages, may be utilized for tree-level calculations of $e^+e^- \rightarrow 4f, 4f\gamma$ with massive fermions. These are GRACE/BASES [16], MADGRAPH/HELAS [17], CompHEP [18] (squared matrix element technique), WPHACT [19], NEXTCALIBUR [20] (initial state radiation photons generated via the structure function approach), HELAC/PHEGAS [21] (recursive Dyson–Schwinger equation approach), WRAP [22] (ALPHA [23] algorithm), O'Mega/WHIZARD [24] and AMEGIC++ [25]. Most of the codes work on the basis of helicity amplitudes and some use the structure function approach to generate the photons. Except for NEXTCALIBUR, which is specialized to $e^+e^- \rightarrow 4f$, all other programs allow to generate and evaluate amplitudes for other type of processes. For more details and comparisons we refer to the *Four Fermion Working Group Report* [26].

Results obtained with $ee4f\gamma$ reproduce nicely, within one standard deviation, the results for all channels of (1) and (2) in the massless fermion limit of SM that have been surveyed in Table 1 of [8]. Moreover, results on (1) with nonzero fermion masses of [6] that had been obtained with $ee4f\gamma$ have been reproduced with AMEGIC++ [27], except for a discrepancy for the rather unphysical channels containing on-shell $t\bar{t}$ -quark-pair, which could be traced back to a different implementation of the top quark width in the propagator.¹

The main advantage of the present program is that it allows for relatively fast computation cross-sections of both (1) and (2). As fermion masses are not neglected, the cross-sections for channels that do not contain e^+ and/or e^- in the final state can be calculated without any kinematic cuts. A number of options have been implemented in the program which make possible calculation of the cross-sections while switching on and off different subsets of the Feynman diagrams. Besides of a possibility of taking into account solely the electroweak contributions, or switching off the Higgs boson exchange diagrams, the program allows also for a simplified treatment of reactions (1), (2) by utilizing two different narrow width approximations: for W bosons

$$e^+(p_1) + e^-(p_2) \rightarrow W^+(p_{34}) + W^-(p_{56}) \quad (3)$$

$$\rightarrow f_1(p_3) + \bar{f}_2(p_4) + f_3(p_5) + \bar{f}_4(p_6), \quad (4)$$

and for a \bar{t} -quark

$$e^+(p_1) + e^-(p_2) \rightarrow t(p_3) + \bar{t}(p_{456}) \quad (5)$$

$$\rightarrow t(p_3) + \bar{b}(p_4) + f_3(p_5) + \bar{f}_4(p_6). \quad (6)$$

2. Scheme of the calculation

The necessary matrix elements of reactions (1) and (2) are calculated with the helicity amplitude method described in [28] and [4] and phase space integrations are performed with the Monte Carlo (MC) method.

In order to improve the convergence of the MC integration the most relevant peaks of the matrix element squared related to the Breit–Wigner shape of the W , Z , Higgs and top quark resonances as well as to the exchange of a massless photon or gluon have to be mapped away. As it is not possible to find out a single parametrization of the multi-dimensional phase space which would allow to cover the whole resonance structure of the integrand, it is necessary to utilize a multi-channel MC approach [29]. The reader is referred to [30] for more details on the multi-channel MC algorithm, on which the MC integration and event generation is based. In the next two subsections we collect phase space parametrizations and mappings of integration variables that are used in the program.

¹ The misprinted entry for σ of $e^+e^- \rightarrow t\bar{b}b\bar{t}$ at $\sqrt{s} = 500$ GeV in Table 1 of [6] should be replaced with 1.2753(7).

2.1. Phase space parametrizations

The basic phase space parametrizations which are used in the program are described in the following. The 4 particle phase space of reaction (1) is parametrized in 2 different ways:

$$d^7\text{Lips} = (2\pi)^{-7} \frac{|\vec{p}_3 + \vec{p}_4|}{4\sqrt{s}} d\cos\theta_{34} ds_{34} ds_{56} dPS_2(s_{34}, m_3^2, m_4^2) dPS_2(s_{56}, m_5^2, m_6^2), \quad (7)$$

$$d^7\text{Lips} = (2\pi)^{-7} \frac{|\vec{p}_3|}{4\sqrt{s}} d\cos\theta_3 ds_{456} ds_{56} dPS_2(s_{456}, s_{45}, m_6^2) dPS_2(s_{56}, m_5^2, m_6^2), \quad (8)$$

where $s = (p_1 + p_2)^2$, $s_{ij\dots} = (p_i + p_j + \dots)^2$ and $dPS_2(p^2, q^2, r^2)$ is a two particle (subsystem) phase space element defined by

$$dPS_2(p^2, q^2, r^2) = \delta^4(p - q - r) \frac{d^3q d^3r}{2q^0 2r^0} = \frac{|\vec{q}|}{4\sqrt{p^2}} d\Omega, \quad (9)$$

with \vec{q} being the momentum and Ω the solid angle of one of the particles (subsystems) in the relative center of mass system, $\vec{q} + \vec{r} = 0$. Momenta $\vec{p}_3 + \vec{p}_4$, \vec{p}_3 and the corresponding polar angles θ_{34} , θ_3 in Eqs. (7) and (8) are defined in the centre of mass system (c.m.s.) frame with a positive direction of the z axis determined by \vec{p}_1 . Using the rotational symmetry with respect to the e^+e^- beam axis, the azimuthal angles of $\vec{p}_3 + \vec{p}_4$ in Eq. (7) and \vec{p}_3 in Eq. (8) are set to zero which reduces the dimension of the phase space integral from 8 to 7. Parametrization (7) covers the kinematic situation where the final state fermions make up two 2-particle subsystems of approximately equal invariant mass. Similarly, the parametrization (8) corresponds to the production of a 1- and 3-particle subsystem with the latter decaying again into 1- and 2-particle subsystems.

The 5 particle phase space of reaction (2) is parametrized in 3 different ways. Parametrizations

$$d^{10}\text{Lips} = (2\pi)^{-10} \frac{E_7}{2} dE_7 d\cos\theta_7 dPS_2(s', s_{34}, s_{56}) ds_{34} ds_{56} \\ \times dPS_2(s_{34}, m_3^2, m_4^2) dPS_2(s_{56}, m_5^2, m_6^2), \quad (10)$$

$$d^{10}\text{Lips} = (2\pi)^{-10} \frac{E_7}{2} dE_7 d\cos\theta_7 dPS_2(s', s_{456}, m_3^2) ds_{456} ds_{56} \\ \times dPS_2(s_{456}, s_{56}, m_4^2) dPS_2(s_{56}, m_5^2, m_6^2), \quad (11)$$

with $s' = (p_1 + p_2 - p_7)^2$ and the photon energy E_7 and the solid angle Ω_7 , being defined in the c.m.s., cover the initial state photon emission. The final state photon emission is covered by the following phase space parametrization

$$d^{10}\text{Lips} = (2\pi)^{-10} \frac{\lambda^{1/2}(s, s_{347}, s_{56})}{8s} \\ \times \frac{1}{8} ds_{347} ds_{56} d\cos\theta_{347} dE_3 dE_7 d\Omega_3 d\phi_{37} dPS_2(s_{56}, m_5^2, m_6^2), \quad (12)$$

where the polar angle θ_{347} of the momentum $\mathbf{p}_3 + \mathbf{p}_4 + \mathbf{p}_7$ is defined in the c.m.s.; the energy E_3 and spherical angle Ω_3 of the final state particle that radiates a photon, the photon energy E_7 and the azimuthal angle ϕ_{37} with respect to the momentum of the radiating particle are defined in the frame where $\mathbf{p}_3 + \mathbf{p}_4 + \mathbf{p}_7 = 0$. Parametrizations (10)–(12) are also used with different permutations of the external particle momenta and we have made use of the rotational symmetry with respect to the beam axis in order to reduce the dimension of integration from 11 to 10.

The invariant masses $s_{ijk\dots}$ in Eqs. (7)–(12) should, if possible, correspond to the virtuality of the propagators of the gauge bosons, Higgs boson and/or top quarks. The peaks related to the propagators are then smoothed out by performing suitable mappings [30].

2.2. Collinear, soft and quasi-real photons

This subsection is dedicated to the issue of collinear, soft and quasi-real photons that is of particular relevance for the precision of phase space integration which is performed in the program.

The strong collinear peaking behaviour of the squared matrix element of reaction (2) corresponding to the radiation off the initial state positron

$$1/[(p_1 - p_7)^2 - m_e^2] \cong 1/[E_7(1 - \beta \cos \theta_7)] \quad (13)$$

is eliminated by the mapping

$$\cos \theta_7 = \frac{1}{\beta_e} [1 - (1 + \beta_e)/r_e^x], \quad (14)$$

with $r_e = (1 + \beta_e)/(1 - \beta_e)$ and $\beta_e = \sqrt{1 - 4m_e^2/s}$ being the velocity of the electron in the c.m.s. Similarly, the collinear peaking related to the radiation off the initial state electron

$$1/[(p_2 - p_7)^2 - m_e^2] \cong 1/[E_7(1 + \beta \cos \theta_7)] \quad (15)$$

is dealt with the mapping

$$\cos \theta_7 = \frac{1}{\beta_e} ((1 - \beta_e)r_e^x - 1). \quad (16)$$

The $\sim 1/E_7$ peaking of the bremsstrahlung photon spectrum of Eqs. (13) and (15) is eliminated by the mapping

$$E_7 = E_7^{\min} (E_7^{\max}/E_7^{\min})^x, \quad (17)$$

where E_7^{\min} and E_7^{\max} are the lower and upper limit of the photon energy E_7 .

The collinear and soft photon peaking corresponding to radiation off a final state fermion, e.g., $f_2(p_4)$, of reaction (2)

$$1/[(p_4 + p_7)^2 - m_4^2] \sim 1/(p_4 \cdot p_7) \sim 1/(C_3 - E_3) \quad (18)$$

is mapped away with the following mapping

$$E_3 = C_3 - (C_3 - m_3) \left(\frac{C_3 - E_3^{\max}}{C_3 - m_3} \right)^x, \quad (19)$$

where $C_3 = \sqrt{s_{347}}/2 + (m_3^2 - m_4^2)/(2\sqrt{s_{347}})$, with $s_{347} = (p_3 + p_4 + p_7)^2$. In Eqs. (13)–(19) and in Eqs. (24)–(28) below, x denotes a random variable uniformly distributed in the interval $[0, 1]$.

In the soft photon limit, the integration over the photon phase space can be performed analytically

$$\begin{aligned} |d\sigma_\gamma|_{|\mathbf{p}_7| < \omega} = & -\frac{1}{(2\pi)^3} \int_{|\mathbf{p}_7| < \omega} \frac{d^3 p_7}{2E_7} \left(g_{\gamma 1} \frac{p_1}{p_1 \cdot p_7} - g_{\gamma 2} \frac{p_2}{p_2 \cdot p_7} + g_{\gamma 3} \frac{p_3}{p_3 \cdot p_7} - g_{\gamma 4} \frac{p_4}{p_4 \cdot p_7} \right. \\ & \left. + g_{\gamma 5} \frac{p_5}{p_5 \cdot p_7} - g_{\gamma 6} \frac{p_6}{p_6 \cdot p_7} \right)^2 d\sigma_0 = - \sum_{i,j=1}^5 g_{\gamma i} g_{\gamma j} I_{ij}^\omega, \end{aligned} \quad (20)$$

where $g_{\gamma i}$ denote the SM couplings of the photon to i th fermion flavour. The bremsstrahlung integrals I_{ij}^ω , which are defined by

$$I_{ij}^\omega = \frac{1}{(2\pi)^3} \int_{|\mathbf{p}_7| < \omega} \frac{d^3 p_7}{2E_7} \frac{p_i \cdot p_j}{(p_i \cdot p_7)(p_j \cdot p_7)}, \quad (21)$$

for $i \neq j$ may be found in Section 7 of Ref. [31]. For $i = j$ we have

$$I_{ii}^{\omega} = \ln \frac{2\omega}{m_{\gamma}} - \frac{1}{\beta_i} \ln \frac{1 + \beta_i}{1 - \beta_i}, \quad (22)$$

where β_i is the velocity of the radiating particle in the c.m.s. and m_{γ} denotes a fictitious mass of the photon that has been introduced in order to regularize the infrared divergence. Splitting up bremsstrahlung reaction (2) into the hard and soft photon parts may serve as a test of the quality of integration over the real photon phase space. The inclusive bremsstrahlung cross-section including both the soft and hard photon parts should be independent of the soft photon energy cut ω . This independence has been successfully tested for different channels of (2) in [4–6]. The user may repeat such tests on his own by running the program with `isoft = 1` and `ihard = 1` with different values of `ecut = \omega`. See Section 3 for explanation of the notation.

Particular attention has also been paid to the channels of (1) and (2) that include a light fermion pair, or an electron and/or a positron in the final state. Matrix elements of such reactions develop semisingularities at phase space regions where the virtuality of the intermediate photon approaches the squared invariant mass of the light fermion pair, or if the photon exchanged in the t -channel becomes quasi-real, as may happen in case of the final state electron and/or positron being hardly scattered.

The decay of the intermediate photon into a light fermion–antifermion pair of invariant mass $s_{f\bar{f}} = p_{f\bar{f}}^2 = (p_f + p_{\bar{f}})^2$ is accounted for by the following mapping of the corresponding $\sim 1/s_{f\bar{f}}$ pole in the Feynman propagator

$$s_{f\bar{f}} = s_{f\bar{f}}^{\min} (s_{f\bar{f}}^{\max}/s_{f\bar{f}}^{\min})^x, \quad (23)$$

with $s_{f\bar{f}}^{\min} = (m_f + m_{\bar{f}})^2$ and $s_{f\bar{f}}^{\max}$ equal to the kinematic upper limit that is computed in the program for any specific phase space parametrization. If this virtual photon becomes quasi-real it is preferably emitted in the direction parallel to initial beams. The cosine of its angle with respect to the beam is then generated with

$$\cos\theta = y - (y + 1)r^x, \quad (24)$$

in case it is parallel to the e^+ beam, or according to the formula

$$\cos\theta = (y - 1)r^{-x} - y, \quad (25)$$

if it is parallel to the e^- beam. In Eqs. (24) and (25),

$$y = \frac{\sqrt{s'} E_{f\bar{f}} - s_{f\bar{f}}}{\sqrt{s'} |\vec{p}_{f\bar{f}}|} \quad \text{and} \quad r = \frac{y - 1}{y + 1}, \quad (26)$$

with $E_{f\bar{f}} = (s_{f\bar{f}} + |\vec{p}_{f\bar{f}}|^2)^{1/2}$; $s' = s$ for reaction (1) and $s' = (p_1 + p_2 - p_7)^2$ for reaction (2).

The pole corresponding to the exchange of a t -channel photon is mapped away by generating cosine of the final state electron angle θ_{e^-} according to

$$\cos\theta_{e^-} = (y - 1)r^x - y, \quad (27)$$

and cosine of the final state positron angle θ_{e^+} with

$$\cos\theta_{e^+} = y - (y + 1)/r^x. \quad (28)$$

In Eqs. (27) and (28), $r = (y + 1)/(y - 1)$ and $y = (E_i E_f - m_e^2)/(|\vec{p}_i||\vec{p}_f|)$, with E_i and \vec{p}_i (E_f and \vec{p}_f) being the energy and momentum of e^- or e^+ in the initial (final) state.

For the charged current reactions, the integration over $\cos\theta_{e^{\pm}}$ can be performed to the very kinematic limit corresponding to

$$t_0 = -m_e^2(E_i - E_f)^2/(E_i E_f). \quad (29)$$

However, for some neutral current channels of reactions (1), (2), in particular those including another light fermion–antifermion pair in addition to e^+e^- in the final state, it may lead to a quasi-real pole $\sim 1/t$ in the photon propagator, corresponding to e^+ or e^- passing from the initial to final state with almost no interaction. This singularity can be avoided by imposing a cut on the angle of the final state electron or positron with respect to the beam.

3. Description of the program

ee4f γ is a package written in FORTRAN 90. It consists of 93 files including a makefile. They are stored in one working directory. The user should specify the physical input parameters in module `inprms.f` and select a number of options in the main program `ee4fg.f`.

3.1. Program input

The default values of the input parameters and options used in the program are those specified below.

3.1.1. Physical parameters

The initial physical parameters to be specified in module `inprms.f` are the following.

Boson masses and widths (GeV):

- $m_W = 80.423$ GeV, the W mass,
- $\Gamma_W = 2.118$ GeV, the W width,
- $m_Z = 91.1876$ GeV, the Z mass,
- $\Gamma_Z = 2.4952$ GeV, the Z width,
- $m_H = 115.0$ GeV, the Higgs mass,
- $\Gamma_H = 0$ GeV, the Higgs width. If $\Gamma_H = 0$, then the Higgs width is calculated according to the lowest order prediction of the SM.

The electroweak (EW) mixing parameter $\sin^2\theta_W$ is then calculated from

$$\sin^2\theta_W = 1 - m_W^2/m_Z^2$$

with

- $m_W^2 = m_W^{**2}$ and $m_Z^2 = m_Z^{**2}$ in the fixed width scheme,
- $m_W^2 = m_W^{**2} - i*m_W*\Gamma_W$ and $m_Z^2 = m_Z^{**2} - i*m_Z*\Gamma_Z$ in the complex mass scheme.

Coupling constants:

- $\alpha^{-1} = 137.03599976$, the inverse of the fine structure constant in the Thomson limit,
- $G_F = 1.16639 \times 10^{-5}$ GeV $^{-2}$, the Fermi coupling constant,
- $\alpha_s = 0.1172$, the strong coupling constant at scale m_Z .

The inverse of the fine structure constant at scale m_W , $\alpha^{-1}(m_W)$, is then calculated from

$$\alpha^{-1}(m_W) = 4.44288293815837 / (2 * \sin^2\theta_W * G_F * m_W^{**2}).$$

Fermion masses and widths:

- $m_e = 0.510998902$ MeV, $g_{me} = 0$ MeV, for the electron,
- $m_\mu = 105.658357$ MeV, $g_{m\mu} = 0$ MeV, for the muon,
- $m_\tau = 1.77699$ GeV, $g_{m\tau} = 0$ GeV, for a tau lepton,
- $m_u = 5$ MeV, $g_{mu} = 0$ MeV, for an up quark,
- $m_d = 9$ MeV, $g_{md} = 0$ MeV, for a down quark,
- $m_c = 1.3$ GeV, $g_{mc} = 0$ GeV, for a charm quark,
- $m_s = 150$ MeV, $g_{ms} = 0$ MeV, for a strange quark,
- $m_t = 174.3$ GeV, $g_{mt} = 1.5$ GeV, for a top quark,
- $m_b = 4.4$ GeV, $g_{mb} = 0$ GeV, for a bottom quark.

No. of colors and conversion constant:

- $ncol = 3$, the number of colors,
- $convc = 0.389379292 \times 10^{12}$ fb GeV², a conversion constant.

3.1.2. Main options

The following main options should be selected in the main program `ee4fg.f`.

- The number of different center of mass (CMS) energies ne
 $ne = 1$. *Recommended if unweighted events are to be generated.*
- The actual values of the CMS energies in the array `aecm` of size ne :
`aecm = (/500.d0/)`.
- The final state of (1) by selecting a value of `ipro` corresponding to a specific channel from the list contained in the file, e.g.,
`ipro = 1`
corresponds to $e^+e^- \rightarrow u\bar{d}\mu^- \nu_\mu^-$. The bremsstrahlung is then calculated for the same channel.
- Whether or not to calculate the Born cross-section, `iborn = 1` (yes) / else (no), with `ncall0` calls to the integrand
`iborn = 1`
`ncall0 = 100000`. *Recommended No. of calls is a few millions.*
- Scan the Born cross-section with `nscan0` calls, `iscan0 = 1` (yes) / else (no), in order to find the dominant kinematic channels, adjust integration weights and find out the maximum value of the cross-section
`iscan0 = 1`. *This option is strongly recommended.*
`nscan0 = 2000`. *A few thousands is recommended.*
- Whether or not to calculate the soft bremsstrahlung cross-section, `isoft = 1` (yes) / else (no), with `ncalls` calls to the integrand
`isoft = 1`
`ncalls = 100000`. *Recommended No. of calls is a few millions.* The integration weights are the same as those used for the Born cross-section.
- Whether or not to calculate the hard bremsstrahlung cross-section, `ihard = 1` (yes) / else (no), with `ncallh` calls to the integrand
`ihard = 1`
`ncallh = 100000`. *Recommended No. of calls is a few millions.*
- Scan the hard bremsstrahlung cross-section with `nscanh` calls, `iscanh = 1` (yes) / else (no)
`iscanh = 1`. *This option is strongly recommended.*
`nscanh = 2000`. *A few thousands is recommended.*
- Generate the unweighted events or not, `imc = 1` (yes) / else (no)?
`imc = 0`.

No standard event record is used. If `imc = 1`, then the final state particle momenta of the accepted unweighted events are printed in file `events.dat`.

- Include the Higgs boson exchange, `ihiggs = 1` (yes) / else (no)?
`ihiggs = 1`
- Calculate the electroweak contributions only, `iew = 1` (yes) / else (electroweak + QCD Feynman diagrams)?
`iew = 0`.
- Whether or not to calculate approximated cross-section of (1) `itopa = 1` (yes) / else (no), with `ncalla` calls to the integrand, relevant only for top quark pair production with an on-shell top quark
`itopa = 0`
`ncalla = 20000`. *Recommended No. of calls is a few hundred thousands.*
- Whether or not to calculate approximated cross-section of (3) `iwwa = 1` (yes) / else (no), relevant only for *W*-pair production
`iwwa = 0`
This uses an analytic formula and does not require MC integration.
- Choose the scheme: `ischeme = 1` (complex mass scheme) / else (fixed width scheme)
`ischeme = 0`
- If `ischeme = 1`, then should `alpha_W` be complex (`iaplw = 1`) or real (`ialpw = 0`)?
`ialpw = 0`.

3.1.3. Auxiliary options

- Choose gauge: `iarbg = 0` (unitary gauge) / `1` (arbitrary linear gauge), relevant only for the nonradiative top quark production
`iarbg = 0`
- If `iarbg = 1`, then choose gauge parameters ($\xi \geq 10^{16}$ corresponds to the unitary gauge):
`ksia = 1.d0`
`ksiz = 1.d16`
`ksiw = 1.d16`
- Specify the soft photon energy cut:
`ecut = 1.d0`
- Impose cuts, `icuts > 0` (yes) / else (no)?
`icuts = 1`
- If `icuts = 1`, then specify the kinematic cuts:
 - `ctlb = 0.985`—cosine of the charged lepton-beam angle,
 - `ctll = 1`—cosine of the angle between charged leptons,
 - `ctlq = 1`—cosine of the charged lepton-quark angle,
 - `ctgb = 0.985`—cosine of the photon-beam angle,
 - `ctgl = cos(5./180*pi)`—cosine of the photon-charged lepton angle,
 - `ctgq = cos(5./180*pi)`—cosine of the photon-quark angle,
 - `ecutg = ecut GeV`—minimum hard photon energy,
 - `ecutl = 5 GeV`—minimum charged lepton energy,
 - `ecutq = 0 GeV`—minimum quark energy,
 - `mqq = 10 GeV`—minimum invariant mass of a quark pair,
 - `mll = 0 GeV`—minimum invariant mass of a charged lepton pair,
- If `icuts > 1`, then specify the kinematic cuts on a final state electron or positron:
 - `theb = 5`—minimum angle of a final state electron and/or positron with respect to the beam.
- Calculate distributions, `idist = 1` (yes) / `0` (no)?
`idis = 0`

- If `idis = 1`, then specify parameters of the distributions:
`nbs = (/ n1, ..., n8/)`—numbers of bins in each distribution.
 The corresponding lower and upper bounds of the distribution arguments
`xmin = (/ x11, ..., x18/)`—lower bounds,
`xmax = (/ xu1, ..., xu8/)`—upper bounds,
 must be specified below in the same file. Constants `xli`, `xui` should be of type `real(8)` and `ni` of type `integer`. The number of desired distributions and the maximum number of bins, `nbmax = max{n1, ..., n8}` should be specified in a module `distrib.f`.
- The maximum of the fully differential cross-section `crmax`, relevant only if `iscan0 = 0` or `iscanh = 0`, `crmax = 1000`.

3.2. Routines of `ee4fg`

The main program `ee4fg`, each subroutine, function or module are located in a file named the same way as the routine itself, except for subroutines `eev`, `eve` and `vee`, contained in `gwwv.f`, and functions: `srr`, `src` and `scc`, contained in `dotprod.f`. The package consists of the following routines.

- SUBROUTINE `boost`—returns a four vector boosted to the Lorentz frame of velocity $-\mathbf{v}$.
- SUBROUTINE `cancuts`—checks if generated particle momenta satisfy kinematic cuts.
- SUBROUTINE `carlos`—the MC integration routine.
- SUBROUTINE `couplsm`—returns SM couplings.
- FUNCTION `cross`—calculates cross-sections of (1) and (2).
- FUNCTION `crosstopa`—calculates approximated cross-section of (1) for top quark pair production with an on-shell top quark in the narrow anti-top width approximation.
- The MAIN PROGRAM `ee4fg`.
- SUBROUTINE `cseeww`—returns the total unpolarized cross-section of on-shell W -pair production.
- SUBROUTINE `cutscrit`—specifies the class of a process (1), i.e., whether the process is purely leptonic, semileptonic, hadronic, etc.
- MODULE `cutvalues`—contains values of kinematic cuts.
- FUNCTION `ddilog`—calculates the real part of a dilogarithm for real arguments.
- MODULE `distrib`—contains parameters of distributions.
- MODULE `drivec`—contains driving flags and some kinematic variables.
- SUBROUTINE `eeee`—returns a contraction of a quartic gauge boson coupling with four complex four vectors.
- FUNCTION `eee`—returns a contraction of a triple gauge boson coupling with three complex four vectors.
- SUBROUTINE `eeff1`—returns the squared matrix element averaged over initial spins and summed over final spins and colors of $e^+e^- \rightarrow f\bar{f}$.
- SUBROUTINE `eeffffcc1a`—returns the squared matrix element averaged over initial spins and summed over final spins and colors of (1) for a charged current processes, $i\text{proc} < 11$ or $14 \leq i\text{proc} < 21$, and the matrix element squared in the double W resonance approximation in arbitrary linear gauge.
- SUBROUTINE `eeffffcc1`—returns the same as `eeffffcc1a` but in the unitary gauge.
- SUBROUTINE `eeffffcc2`—returns the squared matrix element averaged over initial spins and summed over final spins and colors of (1) for the mixed charged and neutral current processes, $11 \leq i\text{proc} < 14$. These matrix elements can be also obtained with `eeffffnc3`.
- SUBROUTINE `eeffffcc3`—returns the same as `eeffffcc2` for $e^+e^- \rightarrow \bar{\nu}_e \nu_e e^+e^-$, $i\text{proc} = 21$.
- SUBROUTINE `eeffffgid1`—returns the squared matrix element averaged over initial spins and summed over final spins and colors of the neutral current processes (2) containing identical leptons, $401 \leq i\text{proc} < 406$.

- SUBROUTINE `eeffffgid2`—returns the same as `eeffffgid1` for the neutral current processes (2) containing identical quarks, $406 \leq \text{iproc} < 412$.
- SUBROUTINE `eeffffgid3`—returns the same as `eeffffgid1` for the neutral current processes (2) containing identical electron neutrinos, $\text{iproc} = 412$.
- SUBROUTINE `eeffffgnc1`—returns the squared matrix element averaged over initial spins and summed over final spins and colors of the neutral current processes (2) containing neither virtual gluons nor an external electron, $21 \leq \text{iproc} < 201$.
- SUBROUTINE `eeffffgnc2`—returns the same as `eeffffgnc1` for the neutral current processes (2) not containing virtual gluons, $201 \leq \text{iproc} < 211$.
- SUBROUTINE `eeffffgnc3`—returns the same as `eeffffgnc1` for the neutral current processes (2) containing $\bar{\nu}_e \nu_e$ -pair, $211 \leq \text{iproc} < 301$.
- SUBROUTINE `eeffffgnc3`—returns the same as `eeffffgnc1` for the mixed charged and neutral current processes (2), $301 \leq \text{iproc} < 401$.
- FUNCTION `eeffffgs`—returns the soft bremsstrahlung correction of Eq. (20) for any channel of (1).
- SUBROUTINE `eeffffid1`—the same as `eeffffgid1` for (1).
- SUBROUTINE `eeffffid2`—the same as `eeffffgid2` for (1).
- SUBROUTINE `eeffffid3`—the same as `eeffffgid3` for (1).
- SUBROUTINE `eeffffnc1`—the same as `eeffffgnc1` for (1).
- SUBROUTINE `eeffffnc2`—the same as `eeffffgnc2` for (1).
- SUBROUTINE `eeffffnc3`—the same as `eeffffgnc3` for (1).
- SUBROUTINE `eeffffnc4`—the same as `eeffffgnc4` for (1).
- SUBROUTINES `eev`, `eve` and `vee` (contained in `gwwv.f`)—return contractions of a triple gauge boson coupling with two polarization vectors leaving, respectively, the third, second and first Lorentz index uncontracted.
- SUBROUTINE `fefefef`—returns matrix elements of the form $\bar{u}_1 \not{\Gamma}_1 S_{F1} \not{\Gamma}_2 S_{F2} \not{\Gamma}_3 u_2$, where Γ_i , $i = 1, 2, 3$, and S_{Fj} , $j = 1, 2$, denote the SM gauge boson-fermion coupling and Feynman propagator of a fermion, respectively.
- SUBROUTINE `fefefefh`—returns the same as `fefefef` including contributions of the Higgs boson exchange.
- SUBROUTINE `fefef`—returns matrix elements of the form $\bar{u}_1 \not{\Gamma}_1 S_{F1} \not{\Gamma}_2 u_2$.
- SUBROUTINE `fefefh`—returns the same as `fefef` including contributions of the Higgs boson exchange.
- SUBROUTINE `fef`—returns matrix elements of the form $\bar{u}_1 \not{\Gamma}_1 u_2$.
- SUBROUTINE `fefh`—returns the same as `fef` including contributions of the Higgs boson exchange.
- SUBROUTINE `ffvgnew`—returns four vectors representing contributions resulting from an attachment of a photon to the SM gauge boson-fermion vertex including a possible Higgs boson exchange contribution.
- SUBROUTINE `fhf`—returns matrix elements $g_{Huu} \Delta_H \bar{u} u$, where g_{Huu} is the Higgs boson coupling to a fermion represented by spinor u and Δ_H is the Feynman propagator of the Higgs boson.
- SUBROUTINE `fsfa`—returns matrix elements $g_{Suu} \Delta_S(\xi) \bar{u} u$, where g_{Suu} is the scalar boson coupling to fermions represented by spinors u_1, u_2 and $\Delta_S(\xi)$ is the Feynman propagator of a scalar boson in an arbitrary linear gauge.
- SUBROUTINE `fsf`—returns the same as `fsfa` in the unitary gauge.
- SUBROUTINE `fvfa`—returns a set of four vectors of the form $\bar{u}_1 \gamma_\nu (g_V^{(-)} P_- + g_V^{(+)} P_+) u_2 D_V^{\nu\mu}(\xi)$, where u_1, u_2 are fermion spinors, $P_\pm = (1 \pm \gamma_5)/2$, are chirality projectors, $g_V^{(\pm)}$ are the gauge boson-fermion couplings of definite chirality and $D_V^{\nu\mu}(\xi)$ is the propagator of a gauge boson V in arbitrary linear gauge.
- SUBROUTINE `fvf`—returns the same as `fvfa` in the unitary gauge.
- MODULE `gaugepar`—contains driving flags and gauge parameters for calculations in arbitrary linear gauge.
- MODULE `genpv12`—contains matrix elements representing the photon and Z boson coupling to initial state fermions.

- MODULE `genpv12g`—contains matrix elements representing the photon and Z boson coupling to initial state fermions including photon radiation from initial lines.
- MODULE `helproc`—contains numbers of possible helicity states of external particles.
- SUBROUTINE `inipart`—calculates content of modules `genpv12` and `genpv12g`.
- MODULE `inprms`—contains initial input parameters.
- FUNCTION `iomega`—calculates the soft bremsstrahlung phase space integrals (21) and (22).
- SUBROUTINE `kineeff1`—returns the four momenta, phase space normalization and flux factor for a $2 \rightarrow 2$ process in the CMS. The phase space is parametrized according to Eq. (9).
- SUBROUTINE `kineeffff`—returns the four momenta, phase space normalization and flux factor for a $2 \rightarrow 4$ process in the CMS. The phase space is parametrized according to Eq. (7) or (8), dependent on selected options.
- SUBROUTINE `kineeffffg`—returns the same as `kineeffff` for a $2 \rightarrow 5$ process in the CMS. The phase space is parametrized according to Eq. (10), (11) or (12), dependent on selected options.
- SUBROUTINE `kineetbff`—returns the same as `kineeffff` with the phase space parametrized according to Eq. (8).
- SUBROUTINE `kineetbffg`—returns the same as `kineeffff` with the phase space parametrized according to Eq. (11).
- SUBROUTINE `kinff`—returns the final state four momenta and phase space normalization for a $2 \rightarrow 2$ process in the CMS.
- SUBROUTINE `kinffg`—returns the same as `kinff` for a $2 \rightarrow 3$ process in the CMS.
- FUNCTION `lamsq`—the kinematic lambda function, $\lambda(\sqrt{x}, \sqrt{y}, \sqrt{z})$.
- SUBROUTINE `mateeee`—returns polarized matrix elements of $e^+e^- \rightarrow e^-e^+e^-e^+$.
- SUBROUTINE `mateeeeg`—returns polarized matrix elements of $e^+e^- \rightarrow e^-e^+e^-e^+\gamma$.
- MODULE `mathprms`—contains necessary arithmetical constants.
- SUBROUTINE `matllll`—returns polarized matrix elements of $e^+e^- \rightarrow l\bar{l}l\bar{l}$.
- SUBROUTINE `matllllg`—returns polarized matrix elements of $e^+e^- \rightarrow l\bar{l}l\bar{l}\gamma$.
- SUBROUTINE `matnnnn`—returns polarized matrix elements of $e^+e^- \rightarrow \nu_e\bar{\nu}_e\nu_e\bar{\nu}_e$.
- SUBROUTINE `matnnnng`—returns polarized matrix elements of $e^+e^- \rightarrow \nu_e\bar{\nu}_e\nu_e\bar{\nu}_e\gamma$.
- SUBROUTINE `matqqqq`—returns polarized matrix elements of $e^+e^- \rightarrow q\bar{q}q\bar{q}$.
- SUBROUTINE `matqqqqg`—returns polarized matrix elements of $e^+e^- \rightarrow q\bar{q}q\bar{q}\gamma$.
- MODULE `parproc`—contains parameters of a specific process.
- MODULE `parsm`—contains SM couplings and squared masses.
- SUBROUTINE `parspec`—returns parameters of a specific process (1) and (2), calculates the top quark and Higgs boson width to the lowest order of SM, the branching ratios for the narrow width approximations; initializes weights for the Monte Carlo integration.
- SUBROUTINE `psneeffff`—returns a phase space normalization as that of `kineeffff` for a given set of external particle momenta.
- SUBROUTINE `psneeffffg`—returns a phase space normalization as that of `kineeffffg` for a given set of external particle momenta.
- SUBROUTINE `psneetbff`—returns a phase space normalization as that of `kineetbff` for a given set of external particle momenta.
- SUBROUTINE `psneetbffg`—returns a phase space normalization as that of `kineetbffg` for a given set of external particle momenta.
- SUBROUTINE `recpol`—returns real polarization vectors of a vector boson in the rectangular basis.
- FUNCTIONS `scc`, `src` and `srr` (contained in `dotprod.f`)—return the Minkowski dot product of two complex, real and complex, and two real four vectors.
- SUBROUTINE `spheric`—returns spherical components ps of a four vector p^μ , $ps = (|\mathbf{p}|, \cos\theta, \sin\theta, \cos\phi, \sin\phi)$, with θ and ϕ being a polar and azimuthal angles of momentum \mathbf{p} .

- SUBROUTINE `spinc`—returns the contractions: $p^0 I - \mathbf{p} \cdot \sigma$ and $p^0 I + \mathbf{p} \cdot \sigma$, where $p^\mu = (p^0, \mathbf{p})$ is a complex four vector, I is the 2×2 unit matrix and σ are the Pauli matrices.
- SUBROUTINE `spinornew`—returns helicity spinors in the Weyl representation.
- SUBROUTINE `spinr`—returns the same contractions as in `spinc` for a real four vector p^μ .
- MODULE `topapprox`—contains SM lowest order widths of the top quark in different approximations.
- SUBROUTINE `uep`—returns a set of spinors of the form $\bar{u} \not{\epsilon} (g_V^{(-)} P_- + g_V^{(+)} P_+) S_F$, where S_F is a Feynman propagator of an internal fermion and the remaining notation is the same as in `fvfa`.
- SUBROUTINE `usp`—returns a set of spinors: $\bar{u} (g_S^{(-)} P_- + g_S^{(+)} P_+) S_F$, where S_F is a Feynman propagator of an internal fermion and g_S^\pm are complex chirality couplings of the scalar boson to the fermion represented by spinor u .
- SUBROUTINE `wwidth`—returns the partial width of the W -boson averaged over initial spins and summed over final spins, and colors for hadronic decay modes.

3.3. Run output

A sample of the listing of the test run output is given in an attached file `test0`. It contains a specification of the process, information on the scheme choice, and values of the relevant physical parameters in the very beginning. If cuts have been imposed (`icut = 1`) then there comes information about specified cuts. Then, if `iscan = 1`, results of the initial scan and resulting weights for the actual run are printed. If calculated weights do not add up exactly to 1 the last weight is changed slightly in order to satisfy the normalization condition. This change is completely irrelevant numerically, but the message about it is printed. The final result for the total cross-section is called `Integral`. Its value is printed together with the standard deviation and the actual number of calls used in the calculation. Finally, if `imc = 1` then information on events acceptance efficiency is given, that means a fraction of accepted weight 1 events. The same kind of output is printed for the soft (without initial scan) and hard bremsstrahlung for each value of the CMS energy. In the very end, all the calculated total cross-sections together with the corresponding standard deviations are collected in the tabular form.

If the event generation option is switched on, i.e., `imc = 1`, then the corresponding number of unweighted events will be printed out as collections of final state particle four momenta in a separate file called `events.dat`. Whenever a maximum value of the cross-section initially assigned in `ee4fg.f`, or found in the result of the initial scan is overflowed a corresponding message informing about it is printed in `events.dat`. The result of integration is still valid, however, if the program is run as an event generator, it should be rerun. How to proceed in this situation will be described in the next section.

4. Use of the program

Up to now the program has been run only on Unix/Linux platforms. In order to run the program, the user should select a specific name of a FORTRAN 90/95 compiler, desired options and the name of the output file in the `makefile`. The output file is called `test` at present. The program can be then compiled, linked and run by executing a single command

```
make test.
```

The results of the test run should reproduce those contained in files `test0` and `events.dat.0`.

The program can be run as the MC event generator of unweighted events by selecting

```
imc = 1
```

in `ee4fg.f`. It is then recommended to run the program for a single CMS energy and to perform a scan with a relatively large number of calls, `nscan0` or `nscanh`. Attention should be paid to possible messages informing about updates of the maximum weight. In this case the program should be rerun, however, this time without the initial scan. The initial integration weights `aw0` or `aw` in `parspec.f` and the maximum weight `crmax` in `ee4fg.f` should be updated according to the results of the prior scan. At present the efficiency of events acceptance is relatively low. However, in view of a relatively fast performance of the program, this should not be a serious limitation for the user.

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